Hindawi Publishing Corporation
Advances in High Energy Physics
Volume 2014, Article ID 304369, 7 pages
http://dx.doi.org/10.1155/2014/304369

Research Article

Extracting the QCD Cutoff Parameter Using the Bernstein Polynomials and the Truncated Moments

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Received 31 August 2014; Revised 10 October 2014; Accepted 13 October 2014; Published 11 November 2014

Academic Editor: Elias C. Vagenas

Since there are not experimental data over the whole range of x-Bjorken variable, that is, $0 < x < 1$, we are inevitable in practice to do the integration for Mellin moments over the available range of experimental data. Among the methods of analysing DIS data, there are the methods based on application of Mellin moments. We use the truncated Mellin moments rather than the usual moments to analyse the EMC collaboration data for muon-nucleon and WA25 data for neutrino-deuterium DIS scattering. How to connect the truncated Mellin moments to usual ones is discussed. Following that we combine the truncated Mellin moments with the Bernstein polynomials. As a result, Bernstein averages which are related to different orders of the truncated Mellin moment are obtained. These averaged quantities can be considered as the constructed experimental data. By accessing the sufficient experimental data we can do the fitting more precisely. We do the fitting at leading order and next-to-leading order approximations to extract the QCD cutoff parameter. The results are in good agreement with what is being expected.

1. Introduction

Deep inelastic scattering (DIS) provides us with a facility to improve our knowledge for subatomic structure and check the precision of quantum chromodynamic (QCD) theory at the parton level. To do the phenomenological task, it is inevitable for one to use the experimental data, resulting from DIS for parton densities and the nucleon structure functions. To make the calculations more reliable, one is forced to apply the limits for the interval of x-Bjorken variable which are imposed by experimental data. This means that we are not able to use the usual and current definition of Mellin moment for parton densities or nucleon structure functions to do the required calculations. This situation automatically leads us to define and use the truncated moments [1–5]. One can find in [1] the relation which exists between the usual moments and the truncated moments that we also use in this paper.

On the other hand, one can employ the Bernstein polynomials to restrict the computations to the regions where there are available experimental data. In [6] one can find that the Bernstein averages of the nucleon structure function can be written as a linear combination of the moments. The truncated moments depend on a Bjorken variable. This variable is in fact the commencing point of an experimental data interval. This point is related to the lower limit of integration to obtain the truncated moments. These moments depend also on the used energy scale. If we go one step further and do another Mellin transformation we can get rid of any dependence on the Bjorken variable. This yields to us a new moment which is called through this paper "moment of moment." As it was pointed out this quantity can be related to usual moments [1]. So it is possible to use this quantity to construct its Bernstein averages. By changing the order of "moment of moment," it is possible to get more and more Bernstein averages which provide us with more experimental data *dependent input* to do a precise fitting.

The organization of this paper is as follwos. In Section 2 we give a review to do the evolution of Mellin moments in the leading order (LO) and next-to-leading order (NLO) approximations. Section 3 is devoted to how the evolution of truncated moments can be done. We introduce the Bernstein polynomials in Section 4. We find the relation between moment of truncated moment (moment of moment) and the related Bernstein averages in Section 5. We use the relation which was obtained in previous section and do a fitting in Section 6 to extract the QCD cutoff parameter at the LO and NLO approximations. Finally we give our conclusion in Section 7.

2. An Overview of Parton Densities Evolution in Mellin Moment Space

The evolution of parton densities with respect to transferred momentum Q^2 is given by [7–13]

$$\frac{dq(x,t)}{dt} = (p \otimes q)(x,t). \tag{1}$$

In this equation the $\otimes$ symbol is representing a convolution integral so as

$$(A \otimes B)(x) \equiv \int_x^1 \frac{dz}{z} A\left(\frac{x}{z}\right) B(z). \tag{2}$$

In (1), $q(x,t)$ is representing the parton densities and $t = \ln\left(Q^2/\Lambda^2_{\overline{\mathrm{MS}}}\right)$. The $\Lambda^2_{\overline{\mathrm{MS}}}$ is QCD cutoff parameter which is determined by fitting the experimental data. This parameter can appear, for instance, in the renormalized running coupling constant at the NLO approximation which is given by

$$\alpha_s(t) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)} - \frac{4\pi\beta_1}{\beta_0^3} \frac{\ln\left(\ln\left(Q^2/\Lambda^2\right)\right)}{\left(\ln(Q^2/\Lambda^2)\right)^2}. \tag{3}$$

Here β_0 and β_1 are the two universal coefficients of QCD-β function. In (3) we mean $\Lambda = \Lambda_{\overline{\mathrm{MS}}}$. In (1), $p(x,t)$ is the splitting function which has the following expansion:

$$p(x,t) = \frac{\alpha_s(t)}{2\pi} p^0(x) + \left(\frac{\alpha_s(t)}{2\pi}\right)^2 p^1(x) + \cdots, \tag{4}$$

where $p^0(x)$ and $p^1(x)$ are representing the splitting coefficients at the LO and NLO approximations, respectively.

Now let us have a brief review on how we can solve the integrodifferential of DGLAP equation in Mellin moment space. The employed method to get the solution will also be used when we intend to get the solution for truncated Mellin moments.

In this regard we should use the Mellin transformation:

$$q^n(t) = \int_0^1 x^{n-1} q(x,t)\, dx. \tag{5}$$

By doing the Mellin transformation on (1), we will arrive at

$$\frac{dq^n(t)}{dt} = \gamma^n(t)\, q^n(t), \tag{6}$$

where $\gamma^n(t)$ is the anomalous dimension which is being obtained, doing the Mellin transformation on the splitting function. It has the following expansion:

$$\gamma^n(t) = \frac{\alpha_s(t)}{2\pi}\gamma^{0,n} + \left(\frac{\alpha_s(t)}{2\pi}\right)^2 \gamma^{1,n} + \cdots. \tag{7}$$

In order to get the solution of evolution equation in Mellin moment space we need as well to use the QCD β-function which has the following form:

$$\beta(\alpha_s) = \frac{d\alpha_s(t)}{dt} = -\frac{\beta_0}{4\pi}\alpha_s^2(t) - \frac{\beta_1}{16\pi^2}\alpha_s^3(t) + \cdots, \tag{8}$$

and we then get

$$dt = \frac{d\alpha_s(t)}{\beta(\alpha_s)}. \tag{9}$$

Considering (3) and (7) at the LO approximation, we can write

$$\gamma^n(t) = \frac{\alpha_s(t)}{2\pi}\gamma^{0,n}(t), \tag{10}$$

$$\alpha_s(t) \simeq \frac{4\pi}{\beta_0 \ln\left(Q^2/\Lambda^2_{\overline{\mathrm{MS}}}\right)}. \tag{11}$$

Substituting (9), (10), and (11) in (6), we will obtain

$$q^n\left(Q^2\right) = q^n\left(Q_0^2\right)\left(\frac{\alpha_s\left(Q^2\right)}{\alpha_s\left(Q_0^2\right)}\right)^{-(2/\beta_0)\gamma^{0,n}}, \tag{12}$$

which is known as the solution of evolution equation at the LO approximation in the Mellin moment space.

To extend the calculation to the NLO approximation we should do the following. Instead of (9) and (10) we will have

$$dt = \frac{d\alpha_s(t)}{-(\beta_0/4\pi)\alpha_s^2(t) - (\beta_1/16\pi^2)\alpha_s^3(t)}, \tag{13}$$

$$\gamma^n(t) = \frac{\alpha_s(t)}{2\pi}\gamma^{0,n}(x) + \left(\frac{\alpha_s(t)}{2\pi}\right)^2 \gamma^{1,n}(t). \tag{14}$$

Substituting (14) into (6) will take us to the solution of Mellin moment of the parton densities at the NLO approximation:

$$q^n\left(Q^2\right) = q^n\left(Q_0^2\right)\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{-2\gamma^{0,n}/\beta_0} \times \left[\frac{4\pi\beta_0 + \beta_1\alpha_s(Q^2)}{4\pi\beta_0 + \beta_1\alpha_s(Q_0^2)}\right]^{(2\gamma^{0,n}/\beta_0 - 4\gamma^{1,n}/\beta_1)}. \tag{15}$$

Equation (15) can be represented as below, using the exponential and Taylor expansion:

$$q^n\left(Q^2\right) = q^n\left(Q_0^2\right)\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{-2\gamma^{0,n}/\beta_0} \times \left[1 + \frac{\alpha_s\left(Q^2\right) - \alpha_s\left(Q_0^2\right)}{\pi\beta_0}\left(\frac{\beta_1\gamma^{0,n}}{2\beta_0} - \gamma^{1,n}\right)\right]. \tag{16}$$

By accessing the parton densities at initial energy scale, Q_0^2, we can obtain its evolved Mellin moments with respect to

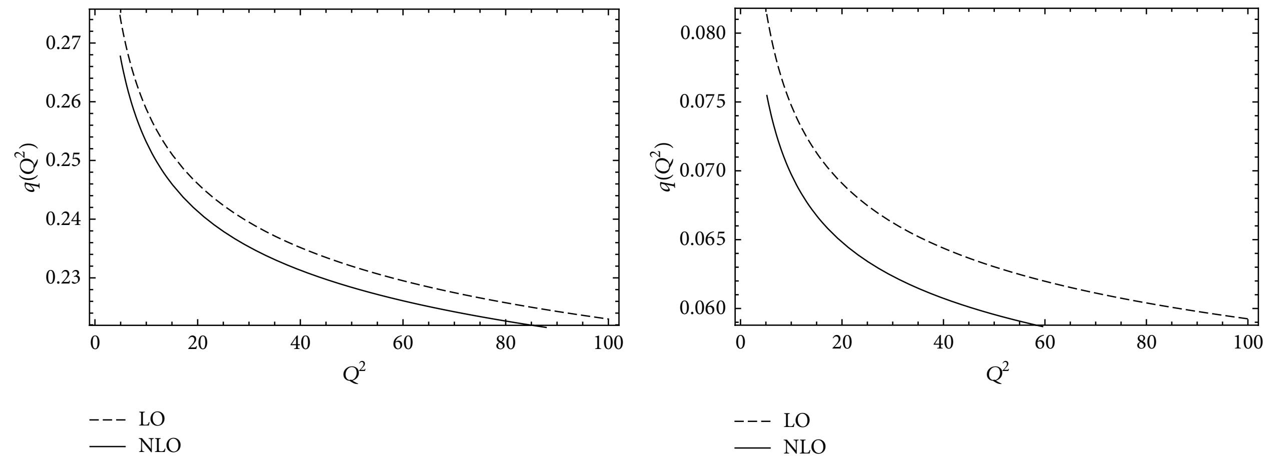

Figure 1: First and second moment of valence parton density versus Q^2 values at the LO and NLO approximations.

different values of Q^2. We plot in Figure 1 the first and second moment of parton densities versus Q^2, using (15). We use the valence parton density in [14] at $Q_0^2 = 0.34\,\mathrm{GeV}^2$ as the input parton density.

In order to obtain the parton densities in the x-Bjorken space, one can use the inverse Mellein transformation as follows:

$$q\left(x, Q^2\right) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dn\, x^{-n} q^n\left(Q^2\right). \qquad (17)$$

Different methods exist to get the parton densities in x space. One of them can be found in [15]. There are different sets of parton densities in x-space. Throughout the whole work we will use the ones of [14].

3. Evolution of the Truncated Mellin Moments

What we did in previous section is based on integration of parton densities over the whole range of x variable, starting from $x = 0$. But it does not practically happen, since in the region where $x \to 0$ the partons do not have any contribution in the carried momentum by nucleon. Furthermore in this region the invariant energy of hadronic system in DIS is raising out to infinity which is physically forbidden. So it is inevitable for one to consider the lower limit of the related integration at the initial value $x_0 = z$ which is different from zero. We should also note that there are not any experimental data for the mentioned region which is again another evidence not to take the lower limit of integration to equal zero. In this case the usual Mellin moment appears in another format which is called truncated Mellin moment [16]:

$$q^n(z,t) = \int_z^1 dx\, x^{n-1} q(x,t). \qquad (18)$$

If we do an integration like the one in (18) on the DGLAP evolution equation, (1), then we obtain

$$\frac{dq^n(z,t)}{dt} = \left(p_n' \otimes q^n\right)(z,t), \qquad (19)$$

where $p_n'(z,t)$ is representing the related splitting function of the truncated moments and is given by

$$p_n'(z,t) = z^n p(z,t). \qquad (20)$$

In order to obtain the solution of (19) we need to get the relation which exists between the usual moments and the truncated moments. Since there is similarity between the evolution equation of usual moments and the truncated moments, that is, (1) and (19), we can use the same technique which is used for (1) to solve the evolution equation for the truncated moments. For this purpose we do Mellin transformation on (19) and call the result as "moment of moment (MM)" which is given by

$$M^{s,n}(t) = \int_0^1 dz\, z^{s-1} q^n(z,t). \qquad (21)$$

By doing the Mellin transformation on (19) we get the following equation:

$$\frac{dM^{s,n}(t)}{dt} = \gamma_{s,n}'(t)\, M^{s,n}(t), \qquad (22)$$

where

$$\gamma_{s,n}'(t) = \int_0^1 dz\, z^{s-1} p_n'(z,t). \qquad (23)$$

Substituting (20) into (23) we achieve a quantity which is analogous to anomalous dimension but it is in fact the Mellin moment of the truncated splitting function and is given by

$$\gamma_{s,n}'(t) = \gamma^{s+n}(t) = \int_0^1 dx\, x^{s+n-1} p(x,t). \qquad (24)$$

The "MM" quantity, given by (21), does not have by itself any usage. In order to employ it in practice, it should be written in terms of the usual Mellin moments which appears as

$$M^{s,n}(t) = \frac{1}{s} q^{s+n}(t) = \frac{1}{s}\int_0^1 dx\, x^{s+n-1} q(t). \qquad (25)$$

Now we can solve evolution equation (19) as we did for usual evolution equation, (1), just by considering this point that during the calculations we should use the "moment of moment" instead of usual moment.

Therefore following the same strategy which leads us to the solution of evolution equation, (1), we can obtain the following results as the solution of the evolution equation for the truncated moments, (19), at the LO and NLO approximations, respectively:

$$M^{s,n}(t) = M^{s,n}(t_0)\left(\frac{\alpha_s(t)}{\alpha_s(t_0)}\right)^{-(2/\beta_0)\gamma^{0,s+n}},$$

$$M^{s,n}(t) = M^{s,n}(t_0)\left(\frac{\alpha_s(t)}{\alpha_s(t_0)}\right)^{-(2/\beta_0)\gamma^{0,s+n}} \times\left[1+\frac{\alpha_s(t)-\alpha_s(t_0)}{\pi\beta_0}\left(\frac{\beta_1}{2\beta_0}\gamma^{0,n+s}-\gamma^{1,n+S}\right)\right]. \quad (26)$$

4. Bernstein Polynomials

The Bernstein polynomials are used as a weight function to get the average of a function, $f(x,t)$ over the $\overline{x}_{m,k} - (1/2)\,\Delta x_{m,k} \le x \le \overline{x}_{m,k} + (1/2)\,\Delta x_{m,k}$ interval [17, 18] in which we can write

$$F_{m,k}(t) \equiv \int_0^1 dx P_{m,k}(x)\, f(x,t), \quad (27)$$

where $F_{m,k}(t)$ is the concerned averaged quantity. The Bernstein polynomials $P_{m,k}(x)$ have the following representation:

$$P_{m,k}(x) = \binom{m}{k} x^k (1-x)^{m-k}. \quad (28)$$

The required quantities which are sited in the concerned interval are given by

$$\begin{aligned} x_{\min} &= \overline{x}_{m,k} - \frac{1}{2}\Delta x_{m,k}, \\ x_{\max} &= \overline{x}_{m,k} + \frac{1}{2}\Delta x_{m,k}; \\ \overline{x}_{m,k} &= \int_0^1 x P_{m,k}(x)\, dx, \\ \overline{x^2_{m,k}} &= \int_0^1 x^2 P_{m,k}(x)\, dx, \\ \Delta x_{m,k} &= \sqrt{\overline{x^2_{m,k}} - \overline{x}_{m,k}^{\,2}}. \end{aligned} \quad (29)$$

To ensure that the integral in (27) is equivalent to this integral over $x_{\min} \le x \le x_{\max}$ we should divide this equation to a normalization factor which is given by $\int_{x_{\min}}^{x_{\max}} dx P_{m,k}(x) dx$.

If we do the Taylor expansion for the Bernstein polynomials we then obtain

$$P_{m,k}(x) = \sum_{l=0}^{m-k} (-1)^l \binom{m}{k}\binom{m-k}{l} x^k x^l, \quad (30)$$

or equivalently

$$P_{m,k}(x) = \frac{(m-k)!\Gamma(m+2)}{\Gamma(k+1)\Gamma(m-k+1)} \times \sum_{l=0}^{m-k}\frac{(-1)^l}{l!(m-k-l)!}x^{k+l}, \quad m > k. \quad (31)$$

Substituting (31) into (27), the averaged quantity, $F_{m,k}(t)$, can be represented by

$$F_{m,k}(t) = \frac{(m-k)!\Gamma(m+2)}{\Gamma(k+1)\Gamma(m-k+1)} \times \sum_{l=0}^{m-k}\frac{(-1)^l}{l!(m-k-l)!}\int_0^1 x^{k+l} f(x,t)\, dx. \quad (32)$$

In terms of the Mellim moments, (32) can be represented by

$$F_{m,k}(t) = \frac{(m-k)!\Gamma(m+2)}{\Gamma(k+1)\Gamma(m-k+1)} \times \sum_{l=0}^{m-k}\frac{(-1)^l}{l!(m-k-l)!}q^{(k+l+1)}(t). \quad (33)$$

5. Truncated Moments and the Brenstein Plolynomails

The truncated moments of parton densities are functions of the carried momentum fraction and the scale energy which are represented by z and Q^2 (or $t = \ln(Q^2/\Lambda^2_{\overline{\mathrm{MS}}})$), respectively. Therefore we can write

$$F^n_{m,k}(Q^2) = \frac{(m-k)!\Gamma(m+2)}{\Gamma(k+1)\Gamma(m-k+1)} \times \sum_{l=0}^{m-k}\frac{(-1)^l}{l!(m-k-l)!}\int_0^1 z^{(k+l+1)-1} q^n(z,Q^2)\, dz, \quad (34)$$

or equivalently (see (21))

$$F^n_{m,k}(Q^2) = \frac{(m-k)!\Gamma(m+2)}{\Gamma(k+1)\Gamma(m-k+1)} \times \sum_{l=0}^{m-k}\frac{(-1)^l}{l!(m-k-l)!}M^{s,n}(Q^2), \quad s = k+l+1, \quad (35)$$

where in (34), $q^n(z,Q^2)$ is the truncated moment which is defined by (18) and $M^{s,n}(Q^2)$ in (35) is its related moment which we called before MM ("momet of moment"). We mean from the order of "MM," the n number in (35). According to (35) the average of the truncated moment is related to a sum over its "MM." The advantage of this equation is that by increasing the order of "MM," that is, $n = 1, 2, 3, \ldots$, we can access more averaged quantities of the related truncated moment. This enable us to construct more data *dependent input* which will improve the precision of the fitting.

6. Extracting the QCD Cutoff Parameter

Here we are intending to show how we can use (35) to extract the QCD cut-off parameter, $\Lambda_{\overline{\mathrm{MS}}}$. First of all we need to determine the digit amounts of m and k for the averaged quantity $F^n_{m,k}(Q^2)$. For this purpose, considering the EMC collaboration data for muon-nucleon DIS scattering [19] and WA25 data for neutrino-deuterium DIS [20] at $Q^2 = 15\,\mathrm{GeV}^2$, we specify the minimum and maximum values of the x variable. Then using (29) we determine the amount of m and k which covers the $x_{\min} \leq x \leq x_{\max}$ interval. We can also determine the m and k by paying attention to the behaviour of Bernstein polynomials with respect to x which leads to similar result. As can be seen from Figure 2 the peaks of the plots are moving by changing the m and k values. We choose the $p_{m,k}(x)$ in which its peak is placed in the interval range of x variable where the population of the experimental data is large. By specifying the indices of the Bernstein polynomials we can calculate the $F^n_{m,k}(Q^2)$ in (35) as the averaged quantity.

By determining m and k we should first obtain $M^{s,n}(Q^2)$ in (35), based on the two theoretical and experimental methods. To follow the theoretical method we go back to Section 3 which gives us the evolution of truncated Mellin moments. To construct the truncated moment, according to (18) we need the parton density function. We get this function from [14] and take the u-valence parton density at $Q^2 = 0.34\,\mathrm{GeV}^2$ as our initial input. We then evolve the truncated moment to energy scale $Q^2 = 15\,\mathrm{GeV}^2$ using (26) which gives us the evolved truncated moments at the LO and NLO approximations, respectively. Therefor considering the determined m and k values at a specified energy scale Q^2, using (35), the $F^n_{m,k}(Q^2)$ as a function of the order of truncated moment, n, can be obtained. We should note that the mathematical property of Bernstein polynomials compels having always $m \geq k$ [21].

Now we illustrate how to obtain the truncated Mellin moment, $M^{s,n}(Q^2)$, experimentally. First we fit a phenomenological function $q(x) = ax^b(1-x)^c$ to the available experimental data for the u-valence quark distribution which are here at $Q^2 = 15\,\mathrm{GeV}^2$. As a result the numerical values of a, b, and c parameters are determined. Now the "MM," that is, $M^{s,n}(Q^2)$, can be obtained experimentally, using (25). Following that, using (35), the averaged quantity $F^n_{m,k}(Q^2)$ is also determined in terms of the linear combination of "MM" at a specified order n while the numerical values of m and k are determined at given value of Q^2.

We can now go to a further step and determine the $\Lambda_{\overline{\mathrm{MS}}}$ parameter. Back to (35) we see that we have the $F^n_{m,k}(Q^2)$ from two stands. On the one hand, we achieve this function theoretically where all quantities are specified except the $\Lambda_{\overline{\mathrm{MS}}}$ parameter which sits on the definition of renormalized coupling constant through "MM" function in (35) (see (26) and (27)). On the other hand, we also access the experimental values of $F^n_{m,k}(Q^2)$ at specified Q^2 which is here $15\,\mathrm{GeV}^2$ while the m ad k are determined. By changing the order of "MM," that is, n, we can construct experimental values for this quantity at different order. Fitting these numerical values to the analytical function of $F^n_{m,k}(Q^2)$ while there is unknown parameter $\Lambda_{\overline{\mathrm{MS}}}$ will yield to us the numerical value for this parameter.

To do more precise fitting we need to increase the produced experimental data. This can be done by changing the numerical values for m and k. For this purpose we use (27) to get the averaged amount of u-valence quark distribution at $Q^2 = 15\,\mathrm{GeV}^2$. This can be done by substituting $f(x,t)$ in (27) with the related phenomenological quark distribution as a function of x-Bjorken variable and Q^2. Now we can change the amount of m and k indices. If the numerical amount of the Bernstein average for the valence density, $q(x,Q^2)$, is near to its experimental value in the concerned interval of x-variable then the chosen values for the m and k are acceptable. We can extend this procedure to find more acceptable values for the m and k. For each value of the m and k we can repeat the procedure described in the two previous paragraphs to find the produced experimental data, $F^n_{m,k}(Q^2)$, at the specified order of n. It is obvious that by changing the order of "MM" we can construct and produce more experimental data for the $F^n_{m,k}(Q^2)$ at the desired m and k values.

Therefore we have some experimental data which contain different values of m and k as well as different numerical values for the order of the "MM. " By doing a global fit we vary in our fitting the amount of m, k, and n. Following that we are able to extract the numerical values for the $\Lambda_{\overline{\mathrm{MS}}}$ at the LO and NLO approximations, respectively. What we got here are $\Lambda^{\mathrm{LO}}_{\overline{\mathrm{MS}}} = 272 \pm 3\,\mathrm{MeV}$ and $\Lambda^{\mathrm{NLO}}_{\overline{\mathrm{MS}}} = 316 \pm 5\,\mathrm{MeV}$, where the four quark active flavours, $n_f = 4$, are used. Let us call these numerical values as the "central values of the extracted results for $\Lambda_{\overline{\mathrm{MS}}}$ ($n_f = 4$)" since they are obtained using the constructed Bernstein averages rather than directly the experimental data. The result is corresponding to what is expected for the QCD cutoff parameter [22, 23]. We should notice that the results of [23] have been obtained at five quark active flavours, $n_f = 5$, by taking the Z-boson mass. But what is reported there is comparable with what we got for the $\Lambda_{\overline{\mathrm{MS}}}$ at the NLO approximation. We should also note that the NLO value of $\Lambda_{\overline{\mathrm{MS}}}$ ($n_f = 5$) extracted in [23] can be compared with our results for the central values of the extracted $\Lambda_{\overline{\mathrm{MS}}}$ ($n_f = 4$) only after transforming the result of [23] from $n_f = 5$ to $n_f = 4$, using the matching condition for quark threshold on the running of the QCD coupling [24, 25]. Meantime the NNLO result of the analysis in [23] may contain additional theoretical uncertainties. For more discussion of this subject see [26]. We can also see that the numerical values for $\Lambda_{\overline{\mathrm{MS}}}$ at the NLO approximation are greater than the LO one which is an acceptable behaviour in all fitting procedure. We depict in Figure 3 the plot of $F^n_{m,k}(Q^2)$ at $Q^2 = 15\,\mathrm{GeV}^2$ for different values of m and k as a function of the MM order, n. As can be seen, the agrement with the produced experimental data is acceptable and the NLO result is better than the LO one.

7. Conclusion

Since there is not experimental data for the whole range of the x-Bjorken variable, one should restrict the limit of integration for the Mellin moment transformation. This will lead us to define the truncated Mellin moments. Based on the

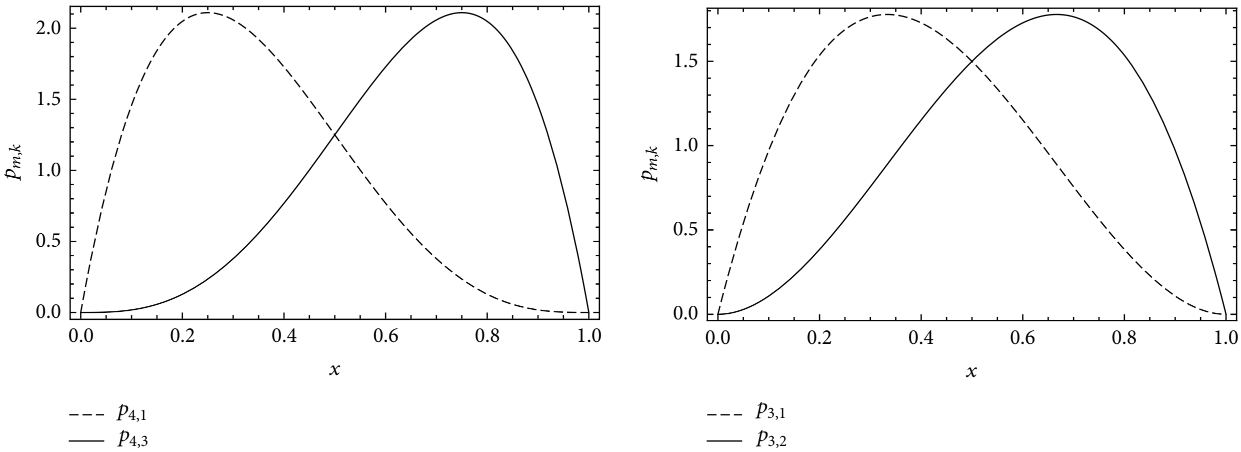

FIGURE 2: The Bernstein polynomials $p_{m,k}(x)$ with respect to x for two different m and k values.

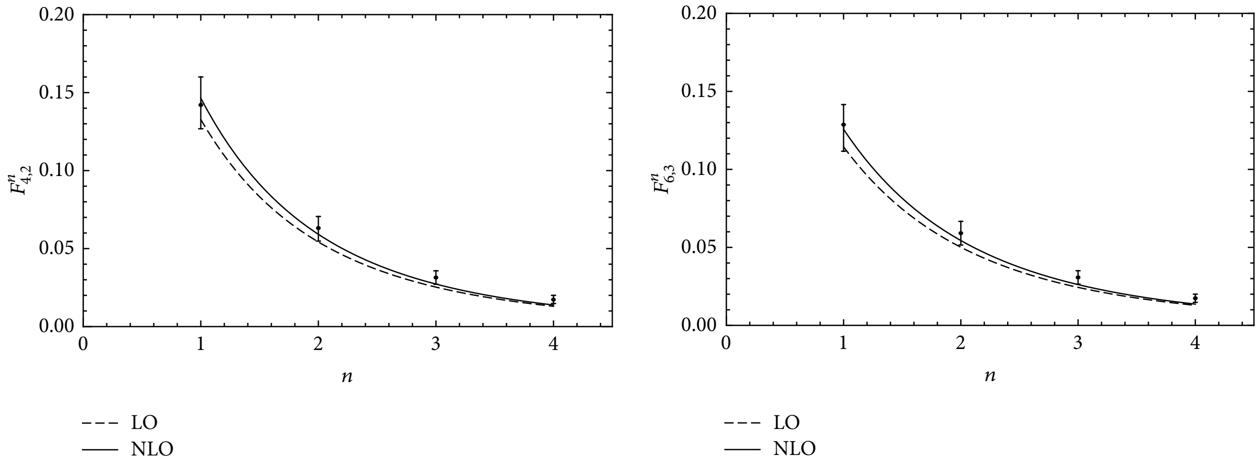

FIGURE 3: The averaged quantity of the truncated Mellin moment as a function of the truncated moment order, n.

DGLAP equation, the evolution of (usual) Mellin moments can be done. Here in this work we were looking to obtain the evolution equations for the truncated moments. The truncated Mellin moments are related to usual moments, using a new quantity which was called moment of moment (MM). In fact this quantity is the Mellin transformation of the truncated moment. "MM" behaves like the usual moments by this difference that contains two indices, (n, s), to indicate the order of the concerned moment. So it can be evolved as usual moment. However in practice we only considered one of these indices, n, as the order of the moment.

On a further step we obtained the average of truncated moment using the Bernstain polynomials. In this case the average of truncated moment can be written in terms of linear combination of the "MMs". This made a possibility to do a fitting and obtain the $\Lambda_{\overline{\text{MS}}}$. For this purpose we used the experimental data for the u-valence of parton density at $Q^2 = 15\,\text{GeV}^2$. We got first the phenomenological parameterizations for this data. Then we obtained its truncated moment and in continuation we would also obtain the moment of this truncated moment which was called "moment of moment." This means that we were able to produce the experimental data for the average of the truncated moment by changing its order.

Two free indices which exist in definition of the Bernstein polynomials can be determined so as to give us the averaged amount for the valence quark densities near the available experimental data over the concerned interval of the x-Bjorken variable. In other words, by suitably choosing m, k, we manage to adjust the region where the average is peaked to that in which we have experimental data.

To do the fitting we made a global fit, using the produced experimental data which was related to the average of truncated moments and extracted the cutoff parameter of QCD at the LO and NLO approximations which corresponded to what one was expecting.

The technique which we used in this paper to do the fitting can be extended to the polarized nucleon structure function. The advantage of this method will appear more in this case since there are few experimental data for the polarized nucleon structure function. But our method makes this possibility to construct more experimental data *dependent* *input* by changing the order of truncated moment. We hope to report about this issue in our further research task.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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